LOYOLA COLLEGE (AUTONOMOUS), CHENNAI – 600 034

M.Sc. DEGREE EXAMINATION – **STATISTICS**

FIRST SEMESTER – NOVEMBER 2015

ST 1820 - ADVANCED DISTRIBUTION THEORY

Date : 03/11/2015 Dept. No. Max. : 100 Marks

Time : 01:00-04:00

SECTION - A

Answer ALL questions. Each carries TWO marks. (10 x 2 = 20 marks)

1. Prove that X, the number of heads obtained when a coin is tossed twice, is a random variable.
2. Give the definition of distribution function and mention its properties.
3. If X has the distribution function

$$F(x) = \begin{cases} 0, & x < 0 \\ \frac{(x+1)}{2}, & 0 \le x < 1 \\ 1, & 1 \le x < \infty \end{cases},$$

   then prove that X is neither discrete nor continuous random variable.
4. Obtain the pdf and mgf of truncated binomial, left truncated at ‘0’.
5. Show that the geometric distribution has lack of memory property.
6. Suppose that $X_1, X_2, \dots, X_n$ are iid non-negative and integer valued random variables. Prove that X_1 is geometric when $X_{(1)} = \text{Min}\{X_1, X_2, \dots, X_n\}$ is geometric.
7. Obtain the pgf and mgf of power-series distribution.
8. Obtain the marginal distributions of X_1 and X_2 , when $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$.
9. Prove that bivariate Poisson distribution has additive property.
10. Write the compound distribution of X, when (i) θ is discrete, (ii) θ is continuous.

SECTION – B

Answer any FIVE questions. Each carries EIGHT marks. (5 x 8 = 40 marks)

11. If the distribution function $F(x) = \begin{cases} 0, & x < 2 \\ \left(\frac{2}{3}\right)x - 1, & 2 \le x < 3 \\ 1, & 3 \le x < \infty \end{cases}$

    then obtain the (i) decomposition of F, (ii) mgf of F.
12. For a truncated Poisson distribution, left truncated at ‘0’, obtain the mean, variance and mgf.
13. Characterize Poisson distribution through pdf.
14. If X_1, X_2 are iid Poisson random variables with parameter λ, then prove that $X_1 \mid X_1 + X_2 = n$ follows $B(n, \frac{1}{2})$.
15. Check whether or not the binomial, Poisson and log-series distributions are power-series distributions.
16. State and establish Skitovitch theorem regarding normal distributions.
17. If X_1, X_2, X_3 are independent normal variables with $E(X_1) = 1$, $E(X_2) = 3$, $E(X_3) = 2$ and $V(X_1) = 2$, $V(X_2) = 2$ and $V(X_3) = 3$, then check whether or not the following pairs are

independent:

(i) $X_1 + X_2$ and $X_1 - X_2$

(ii) $X_1 + X_2 - 2X_3$ and $X_1 - X_2$

(iii) $2X_1 + X_3$ and $X_2 - X_3$.

18. Derive the mgf of inverse Gaussian distribution.

SECTION – C

Answer any TWO questions. Each carries TWENTY marks. (2 x 20 = 40 marks)

19(a) For a power-series distribution, obtain the first cumulant k_1 and the recurrence formula for obtaining r^{th} cumulant k_r. Therefore find k_r for Poisson distribution. (10)

(b) Let $(X_1, X_2) \sim BB(n, p_1, p_2, p_{12})$. Prove that $X_1 \mid X_2 = x_2 \overset{d}{=} U_1 + V_1$, where $U_1 \sim B\left(n - x_2, \frac{p_1}{q + p_1}\right)$, $V_1 \sim B\left(x_2, \frac{p_{12}}{p_2 + p_{12}}\right)$ and U_1 is independent of V_1. (10)

20(a) Obtain the relation among mean, median, and mode of a log-normal distribution. (10)

(b) If $X_1 \sim G(\alpha, p_1)$, $X_2 \sim G(\alpha, p_2)$ and X_1 is independent of X_2, then prove the following:

(i) $X_1 + X_2 \sim G(\alpha, p_1 + p_2)$,

(ii) $X_1 / (X_1 + X_2) \sim$ Beta distribution of first kind,

(iii) $(X_1 + X_2)$ is independent of $(X_1 / (X_1 + X_2))$. (10)

21(a) Prove that $((X - \mu)^2) / (\mu^2 X) \sim \chi^2(1)$, when $X \sim IG(\mu, \lambda)$. (8)

(b) Obtain the conditional distribution of (i) $X_2 \mid X_1 = x_1$, (ii) $X_1 \mid X_2 = x_2$, when $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. (12)

22(a) Obtain the mgf of (X_1, X_2) at (t_1, t_2), when $(X_1, X_2) \sim BVN(\mu_1, \mu_2, \sigma_1^2, \sigma_2^2, \rho)$. (10)

(b) Give the definition of non-central t – distribution. Hence obtain its pdf. (10)